Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **8**, No. 6, pp. 1036-1049

DOI: 10.4208/aamm.2015.m1121
December 2016

New Non-Travelling Wave Solutions of Calogero Equation

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Received 20 May 2015; Accepted (in revised version) 16 November 2015

Abstract. In this paper, the idea of a combination of variable separation approach and the extended homoclinic test approach is proposed to seek non-travelling wave solutions of Calogero equation. The equation is reduced to some $(1+1)$-dimensional nonlinear equations by applying the variable separation approach and solves reduced equations with the extended homoclinic test technique. Based on this idea and with the aid of symbolic computation, some new explicit solutions can be obtained.

AMS subject classifications: 35C09, 35Q53, 68W30

Key words: Variable separation approach, extended homoclinic test approach, non-travelling wave solution.

1 Introduction

Nonlinear evolution equations are related to nonlinear phenomena in nonlinear science such as physics, mechanics, biology and chemistry. To further explain some physical phenomena, seeking exact solutions of nonlinear evolution equations is of great significance. It is well known that the method of variable separation is one of the most universal and efficient means for studying linear partial differential equations (PDEs). Several methods of variable separation for nonlinear PDEs have been suggested, such as the ansatz-based method [1], the formal variable separation approach [2], the functional variable separation approach [3, 4], the derivative-dependent functional variable separation approach [5, 6] and the multi-linear variable separation approach [7–9].

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In this paper, we consider the following $(2+1)$-dimensional breaking soliton equation

$$u_{xt}-4u_xu_{xy}-2u_yu_{xx}+u_{xxxy}=0, \tag{1.1}$$

which was first presented by Calogero and Degasperis [10, 11]. This equation was also constructed by Bogoyavlenskii and Schiff in different ways [12–14]. The equation is used to describe the interaction of a Riemann wave propagating along the y axis with a long wave along the x axis. The Plainlevé property, Darboux covariant Lax pairs, infinite conservation laws, Hamilton structure and the Lax pair of Eq. (1.1) have been discussed by many researchers in [15–17]. The bilinear Bäcklund transformation, nonlinear superposition formula and Wronskian determinant solution have been discussed in [18]. Moreover, a considerable number of exact specific solutions have been developed and can be found in [19–25].

The goal of the present work is to investigate the non-travelling wave solutions for Eq. (1.1) by using the multi-linear variable separation approach combining with the extended homoclinic test approach [26]. First, we apply the method of variable separation to reduce Eq. (1.1) to some $(1+1)$-dimensional nonlinear equation. Then, solving the reduced equation by the extended homoclinic test technique with the aid of Maple, we obtain some new non-travelling wave explicit solutions of Eq. (1.1). These solutions can provide an important practical check on the accuracy and reliability of such integrators.

2 The non-travelling wave solutions

In this section, we employ the method of separation of variables together with the extended homoclinic test technique solving the Calogero Equation (1.1). We assume that the solutions for Eq. (1.1) are of the x-line form

$$u(x,y,t)=\varphi(\xi,t)+q(y,t), \tag{2.1}$$

where $\xi=px+\theta(y,t)$.

Substituting (2.1) into (1.1), one obtains

$$\varphi_{\xi t}+(\theta_t-2pq_y)\varphi_{\xi\xi}-6p\theta_y\varphi_\xi\varphi_{\xi\xi}+p^2\theta_y\varphi_{\xi\xi\xi\xi}=0. \tag{2.2}$$

In order to simplify Eq. (2.2), we assume that

$$\theta_t-2pq_y=0. \tag{2.3}$$

Now we employ the method of separation of variables solving Eq. (2.3).

First we seek the multiplicative separable solution

$$\theta=f(t)g(y), \tag{2.4}$$

where $f(t)$, $g(y)$ are smooth functions to be determined later.