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Implicit DG Method for Time Domain Maxwell's Equations Involving Metamaterials

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Abstract. An implicit discontinuous Galerkin method is introduced to solve the time-domain Maxwell's equations in metamaterials. The Maxwell's equations in metamaterials are represented by integral-differential equations. Our scheme is based on discontinuous Galerkin method in spatial domain and Crank-Nicolson method in temporal domain. The fully discrete numerical scheme is proved to be unconditionally stable. When polynomial of degree at most p is used for spatial approximation, our scheme is verified to converge at a rate of $\mathcal{O}(\tau^2+h^{p+1/2})$. Numerical results in both 2D and 3D are provided to validate our theoretical prediction.

AMS subject classifications: 65M12, 65M60

Key words: Maxwell's equations, metamaterials, fully disctete, DG method, L^2-stability, L^2-error estimate.

1 Introduction

The metamaterials are artificially structured electromagnetic materials. It has some exotic properties, such as negative refractive index and amplification of evanescent waves, which may not be found in nature. Since it was first constructed in 2000, there are a number of works on the study of metamaterials and their applications in different areas.

To our knowledge, the numerical simulation of metamaterials plays an important role in the design of new metamaterials and discovery of new phenomenon of them [5]. Among them, the widely used numerical methods for the simulation of metamaterials are finite difference time domain method (FDTD) [9, 24], finite element method (FEM) [19] and the commercial packages, such as HFSS and COMSOL et al..

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Since the discontinuous Galerkin (DG) method was first proposed in 1973 [23], it has become one of the most popular methods for solving various partial differential equations [2, 11]. Actually, the DG method uses discontinuous piecewise polynomials as both trial and test functions. In this approach, the discontinuities at the element interfaces allow the design of suitable inter-element boundary treatments to obtain highly accurate and stable methods in many difficult situations. It is well known that the DG method has several distinctive advantages, e.g., applicability for non-conforming mesh, high-order accuracy, flexibility in handling material interface and high parallelizability. We refer to the survey papers [1, 4] and the books [6, 11] and their references therein for more details about it.

The DG methods have been investigated for Maxwell's equations in both free space [3, 7, 8, 27] and dispersive media [12, 14, 15, 18, 21, 25, 26] whose permittivity depends on the wave frequency. In [25], Wang et al. introduce a semi-discrete divergence-free DG method for solving Maxwell's equations in dispersive media under a unified framework. It is proved that the convergence rate of the semi-discrete method is $\mathcal{O}(h^{p+1/2})$. Actually the discretization of the spatial domain, leads to a Volterra integro-differential system in time t. Then a continuous Galerkin method is used to solve this reduced system. In [27], Xie et al. develop an unconditionally stable space-time DG method for solving Maxwell's equations in free space and obtain the convergence rate of $\mathcal{O}(\tau^{p+1}+h^{p+1/2})$ in the L^2-norm when the polynomials of degree at most p are used in both temporal and spatial discretization. In [26], Wang et al. extend this space-time DG method to dispersive media and give both the theoretical analysis and numerical examples.

Recently, Li in [16] develop a DG method in space for solving Maxwell's equations in metamaterials and perfectly matched layers with Runge-Kutta method in time. In it several numerical examples were given to show that this method is efficient, but the theoretical analysis is missing. In [20], Li et al. develop a leap-frog type DG method for solving the time-domain Maxwell's equations in metamaterials, and provide both the stability and convergence analysis. In [18], Li et al. develope a leap-frog type DG method for solving the time-domain Maxwell's equations in metamaterials based on an auxiliary differential equations (ADE) method, and prove that, under some CFL condition, this method is stable and convergent.

The main aim of this paper is to develop a new scheme for solving time-domain Maxwell's equations in metamaterials. The model problem is the first-order Maxwell's equations in metamaterials where the dispersive character is taken into account via an integral term. Our scheme is based on discontinuous Galerkin method in spatial domain and Crank-Nicolson method in temporal domain. Then the unconditional stability and convergence rate of $\mathcal{O}(\tau^2+h^{p+1/2})$ are obtained for our scheme. Theoretical results are validated by some numerical examples.

The rest of this paper is organized as follows. In Section 2, we present the governing equations for metamaterials. The fully-discrete scheme is introduced in Section 3. Both the L^2-stability and L^2-error estimate are proved in Section 4. In Section 5, some numerical results are provided to support our theory analysis.