Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **6**, No. 3, pp. 281-298

DOI: 10.4208/aamm.12-m1295
June 2014

A Second-Order Three-Level Difference Scheme for a Magneto-Thermo-Elasticity Model

Hai-Yan Cao, Zhi-Zhong Sun* and Xuan Zhao

Department of Mathematics, Southeast University, Nanjing 210096, China

Received 6 July 2012; Accepted (in revised version) 13 December 2013

Available online 17 April 2014

Abstract. This article deals with the numerical solution to the magneto-thermo-elasticity model, which is a system of the third order partial differential equations. By introducing a new function, the model is transformed into a system of the second order generalized hyperbolic equations. A priori estimate with the conservation for the problem is established. Then a three-level finite difference scheme is derived. The unique solvability, unconditional stability and second-order convergence in L_∞-norm of the difference scheme are proved. One numerical example is presented to demonstrate the accuracy and efficiency of the proposed method.

AMS subject classifications: 65M06, 65M12, 65M12, 78M20, 80M20

Key words: Magneto-thermo-elasticity, conservation, finite difference, solvability, stability, convergence.

1 Introduction

In the past decades, magneto-thermo-elastic theory has been widely applied in acoustics, geophysics, micro electromechanical systems (MEMS). There are some reviews about the classical and generalized theories [1–3]. The generalized thermo-elasticity theories were considered to be more realistic than the conventional theory in dealing with practical problems. Some models have been proposed in order to study the property of the analytical solution by the energy functional and generalized variational principle [4–8]. As is known to all, it is difficult to find the analytical solution for the generalized model. Thus, the numerical solutions are usually obtained by the numerical methods such as finite difference method [9–12], finite element method [13–16] and numerical integration method [17–19]. It should be mentioned that there are few papers concentrating on the

*Corresponding author.
Email: maths@seu.edu.cn (H. Y. Cao), zzsun@seu.edu.cn (Z. Z. Sun), xuanzhao11@gmail.com or xzhao@seu.edu.cn (X. Zhao)

analysis of the established numerical methods. This paper deals with the Green-Naghdi (G-N) model [20] derived by Green and Naghdi [21–23] who provided sufficient basic modifications in the constitutive equations. The model is written as follows:

$$R_M^2\frac{\partial^2 U}{\partial \xi^2}-\frac{\partial \theta}{\partial \xi}=\frac{\partial^2 U}{\partial \eta^2}, \tag{1.1a}$$

$$\frac{\partial^2 \theta}{\partial \eta^2}+\varepsilon_T\frac{\partial^3 \theta}{\partial \xi\partial \eta^2}=c_T^2\frac{\partial^2 \theta}{\partial \xi^2}+\kappa_0\frac{\partial^3 \theta}{\partial \xi^2\partial \eta}, \tag{1.1b}$$

where $U=U(\xi,\eta)$ and $\theta=\theta(\xi,\eta)$ are functions of displacement and temperature, ξ and η are space variable and time variable respectively. The constants R_M^2, ε_T, c_T and κ_0 are dimensionless quantities, where R_M^2 describes the impact of the external magnetic field in the process of thermo-elasticity, ε_T is the thermo-elasticity coupled coefficient, c_T is the wave velocity in G-N model, and κ_0 is the thermal diffusion coefficient. When $\kappa_0\ll c_T^2$, Eq. (1.1b) becomes

$$\frac{\partial^2 \theta}{\partial \eta^2}+\varepsilon_T\frac{\partial^3 \theta}{\partial \xi\partial \eta^2}=c_T^2\frac{\partial^2 \theta}{\partial \xi^2}, \tag{1.2}$$

which corresponds to the thermo-elasticity undamped heat-wave solution in G-N model. For emphasizing the main idea, in this article, we consider the model with the simplified notations in the bounded domain. Then (1.1a) and (1.2) turn into

$$u_{tt}=au_{xx}-v_x,\qquad 0<x<1,\qquad 0<t\le T, \tag{1.3a}$$

$$v_{tt}=cv_{xx}-bu_{xtt},\qquad 0<x<1,\qquad 0<t\le T. \tag{1.3b}$$

Now taking the derivative with respect to t on both sides of the Eq. (1.3a), we have

$$u_{ttt}=au_{xxt}-v_{xt},\quad 0<x<1,\quad 0<t\le T.$$

Let $w=u_t$, then the above equation is equivalent to

$$w_{tt}=aw_{xx}-v_{xt},$$

and (1.3b) can be rewritten as

$$v_{tt}=cv_{xx}-bw_{xt}.$$

In the following we consider the numerical solution of initial boundary value problem for the coupled system:

$$w_{tt}=aw_{xx}-v_{xt}+g_1(x,t),\qquad 0<x<1,\quad 0<t\le T, \tag{1.4a}$$

$$v_{tt}=cv_{xx}-bw_{xt}+g_2(x,t),\qquad 0<x<1,\quad 0<t\le T, \tag{1.4b}$$

$$w(x,0)=\phi_1(x),\quad w_t(x,0)=\phi_2(x),\qquad 0\le x\le 1, \tag{1.4c}$$

$$v(x,0)=\psi_1(x),\quad v_t(x,0)=\psi_2(x),\qquad 0\le x\le 1, \tag{1.4d}$$

$$w(0,t)=\alpha_1(t),\quad w(1,t)=\alpha_2(t),\qquad 0<t\le T, \tag{1.4e}$$

$$v(0,t)=\beta_1(t),\quad v(1,t)=\beta_2(t),\qquad 0<t\le T, \tag{1.4f}$$