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# Buckling of 2D-FG Cylindrical Shells under Combined External Pressure and Axial Compression

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**Abstract.** This paper presents the stability of two-dimensional functionally graded (2D-FG) cylindrical shells subjected to combined external pressure and axial compression loads, based on classical shell theory. The material properties of functionally graded cylindrical shell are graded in two directional (radial and axial) and determined by the rule of mixture. The Euler's equation is employed to derive the stability equations, which are solved by GDQ method to obtain the critical mechanical buckling loads of the 2D-FG cylindrical shells. The effects of shell geometry, the mechanical properties distribution in radial and axial direction on the critical buckling load are studied and compared with a cylindrical shell made of 1D-FGM. The numerical results reveal that the 2D-FGM has a significant effect on the critical buckling load.

**AMS subject classifications**: 74K25, 74G60

**Key words**: Mechanical buckling, 2D-FG cylindrical shell, combined load, classical shell theory, GDQ method.

## 1 Introduction

Functionally graded structures are those in which the volume fractions of two or more materials are varied smoothly and continuously as a function of positions along with certain direction(s) of the structure to achieve a required function. These materials were used for the first time by a group of scientists in Sendai, Japan, in 1984 [1,2]. The gradual change of material properties can be proportional to different applications and working environments.

Mechanical Buckling of a circular cylindrical shell, as a major structure, is being studied for a long time. The buckling behavior of structural members made of homogeneous

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materials subjected to mechanical loads are studied by Brush and Almorth [3]. Vodenitcharova and Ansourian [4] presented the buckling analysis of circular cylindrical shells subjected to a uniform lateral pressure. Based on the first order shear deformation theory (FSDT), Khazaeinejad et al. [5] studied on the buckling of functionally cylindrical shells under external pressure and axial compression.

The most familiar FGM is compositionally graded from one surface to another with a prescribed function. But conventional FGM may not be so effective in some structures since all outer or inner surfaces will have the same composition distributions while in developed machine elements, temperature and load distribution may change in two or more directions [6]. For this reason, we introduced 2D-FGM properties which are dependent bi-directionally. Recently some works have been done on 2D-FGM. Dhaliwal and Singh [7] solved the equations of equilibrium for a non-homogeneous elastic solid under shearing forces. Sobhani Aragh and Hedayati [8] studied the static response and free vibration of two-dimensional functionally graded metal/ceramic open cylindrical shells under various boundary conditions. Pindera and Aboudi studied a coupled higher-order theory for cylindrical structural components with bi-directionally graded microstructures [9]. Also Asgari et al. [10] considered the solution of dynamic analysis of a thick hollow cylinder with finite length made of two-dimensional functionally graded material (2D-FGM) and subjected to the impact of internal pressure.

Numerous methods have been developed and used for studying the buckling of circular cylindrical shells. However, in this study, using the GDQ method, the stability equations and critical buckling loads are obtained. The GDQ method is a global approximate method. In GDQ method the derivative of a function with respect to a coordinate direction can be expressed as a weighted linear sum of all the functional values at all mesh points along that direction and a continuous function can be approximated by high order polynomials in the overall domain [11].

According to the authors' information, there haven't been any investigations on the buckling of 2D-FG cylindrical shells. In this study, the stability of 2D-FG circular cylindrical shells subjected to combined mechanical loads and based on the classical shell theory is presented considering Young's Modulus changes the material in two directional (radial and axial). To express the combination of applied axial compression and external pressure, a load interaction parameter is defined. The critical buckling loads are obtained for variation of the material constitutions, load interaction parameters, aspect ratios and thickness ratios. Comparing studies are presented to validate the present analysis results. Also, the results reveal that the 2D-FGM has significant effect on the critical buckling load.

## 2 Material and methods

Consider a cylindrical shell made of two directional functionally graded material (2D-FGM) of mean radius $R$, thickness $h$, and length $L$ as shown in Figs. 1 and 2. The 2D-FG