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A Spectral Method for Second Order Volterra Integro-Differential Equation with Pantograph Delay

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Abstract. In this paper, a Legendre-collocation spectral method is developed for the second order Volterra integro-differential equation with pantograph delay. We provide a rigorous error analysis for the proposed method. The spectral rate of convergence for the proposed method is established in both L^2-norm and L^∞-norm.

AMS subject classifications: 65R20, 34K28

Key words: Legendre-spectral method, second order Volterra integro-differential equation, pantograph delay, error analysis.

1 Introduction

The paper is concerned with the second order Volterra integro-differential equation with pantograph delay:

$$\begin{aligned} u^{(2)}(x)=&\sum_{j=0}^{1}a_j(x)u^{(j)}(q_jx)+\sum_{j=0}^{1}b_j(x)u^{(j)}(x)\\ &+\sum_{j=0}^{1}\int_0^x k_j(x,s)u^{(j)}(s)ds+g(x), \qquad x\in[0,T], \end{aligned} \tag{1.1}$$

with

$$u(0)=u_0, \qquad u^{(1)}(0)=u_1. \tag{1.2}$$

Here, we denote $u^{(j)}(x)=(d^j/dx^j)u(x)$, $j=0,1,2$. q_j is a given constant and $0<q_j<1$. $a_j(x)$, $b_j(x)$, $g(x)$ are smooth functions on [0,T]. $k_j(x,s)$ is also a smooth function on $D(D:=$

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$\{(x,s): 0\leq s\leq x\leq T\}$) and $u^{(j)}(q_jx)$ is pantograph delay, $j=0,1$. $u(x)$ is an unknown function.

Since Volterra integro-differential equations with pantograph delay arise widely in the mathematical model of physical and biological phenomena, many researchers have developed theoretical and numerical analysis for the related types of equations. We refer the reader to [3, 6, 10] for a survey of early results on Volterra integro-differential equations. More recently, polynomial spline collocation methods were investigated in [16, 19] and homotopy analysis method was used to solve system of Volterra integral equations (see, e.g., [14]). In [20, 21], the authors used spectral collocation methods studying convergence analysis about Volterra integro-differential equations. For pantograph delay differential equations, in [2, 12, 23], the authors researched on these kinds of functions. In [1], spectral method was used to solve $y'(x)=a(x)y(qx)$, but it only analysed the numerical error in the infinity norm.

So far, very few work have touched the spectral approximation to second order Volterra integro-differential equations with pantograph delay. In practice, spectral method has excellent convergence property of exponential convergence rate. In this paper, we will provide a Legendre-collocation spectral method for the second order Volterra integro-differential equation with pantograph delay and analyse the numerical error decay exponentially in both L^2 and L^∞ space norms.

For ease of analysis, we will describe the spectral method on the standard interval $[-1,1]$. Hence, we employ the transformation

$$x=\frac{T}{2}(1+t), \qquad t=\frac{2x}{T}-1.$$

Then the above problem (1.1)-(1.2) becomes

$$\begin{aligned} y^{(2)}(t)=&\sum_{j=0}^{1}A_j(t)y^{(j)}(q_jt+q_j-1)+\sum_{j=0}^{1}B_j(t)y^{(j)}(t)\\ &+\sum_{j=0}^{1}\int_0^{\frac{T}{2}(1+t)}\hat{k}_j(t,s)u^{(j)}(s)ds+G(t), \qquad t\in[-1,1], \end{aligned} \tag{1.3}$$

with

$$y(-1)=u_0, \qquad y^{(1)}(-1)=\Big(\frac{T}{2}\Big)u_1, \tag{1.4}$$

where

$$\begin{aligned} &y(t)=u\Big(\frac{T}{2}(1+t)\Big), && G(t)=\Big(\frac{T}{2}\Big)^2 g\Big(\frac{T}{2}(1+t)\Big),\\ &A_0(t)=\Big(\frac{T}{2}\Big)^2 a_0\Big(\frac{T}{2}(1+t)\Big), && A_1(t)=\Big(\frac{T}{2}\Big)a_1\Big(\frac{T}{2}(1+t)\Big), \end{aligned}$$