Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **9**, No. 4, pp. 861-867

DOI: 10.4208/aamm.2015.m1292
August 2017

The Variational Iteration Method for an Inverse Problem of Finding a Source Parameter

Zongli Ma[1,2,*] and Shumin Li[1]

[1] *School of Mathematical Sciences, University of Science and Technology of China, Hefei, Anhui 230026, China*
[2] *School of Mathematics and Computing Science, Anqing Teacher College, Anqing, Anhui 246011, China*

Received 11 September 2015; Accepted (in revised version) 19 July 2016

Abstract. An inverse problem of determining unknown source parameter in a parabolic equation is considered. The variational iteration method (VIM) is presented to solve inverse problems. The solution gives good approximations by VIM. A numerical example shows that the VIM works effectively for an inverse problem.

AMS subject classifications: 35K05, 35K99, 47J06, 49N45

Key words: Inverse problem, variational iteration method, parabolic equation.

1 Introduction

In this paper, we consider numerical procedures for the following parabolic equation with a time-dependent coefficient

$$u_t=\Delta u+\sum_{j=1}^{n}q_j(t)u_{x_j}+p(t)u(x,t)+f(x,t), \qquad x\in\Omega\subset R^n, \quad 0<t\leq T, \tag{1.1a}$$

$$u(x,0)=\varphi(x), \qquad x\in\Omega, \tag{1.1b}$$

$$u(x,t)=g(x,t), \qquad (x,t)\in\partial\Omega\times[0,T], \tag{1.1c}$$

subject to the additional condition

$$\int_\Omega u(x,t)dx=a(t), \quad t\in[0,T], \tag{1.2}$$

*Corresponding author.
Email: sdmzl@126.com (Z. L. Ma)

where $q_j(t)$, $j=1,\cdots,n$, $f(x,t)$, $\varphi(x)$, $g(x,t)$ and $a(t)$ are given continuous functions and Ω is a bounded domain with continuous boundary. We need to obtain a pair $\{u(x,t),p(t)\}$ from (1.1a)-(1.2).

If we let $u(x,t)$ represent the temperature, this problem can model a heat transfer process with an unknown source parameter $p(t)$. It can be regarded as a source identification problem. The parameter $p(t)$ needs to be determined from the additional condition (1.2).

The condition (1.2) is called the thermal energy measurement. It means that we should measure the thermal energy in every moment to determine the source parameter $p(t)$. The presence of an integral term in a boundary condition can greatly complicate the application of standard numerical techniques such as finite differences, finite elements, spectral methods, etc. It is therefore important to search effective means to deal with it. When one deals with (1.1a)-(1.2) by finite difference schemes, a very natural ideas is to utilize the numerical integration as in [1]. The boundary element method was presented in [2] to discretize the heat equation (1.1a). By means of the fundamental solution and the Green's formula, they converted the original differential equation to boundary integral equation.

The existence, uniqueness and continuous dependence of the solutions to this problem is discussed in [3,4].

The variational iteration method(VIM), which was proposed by He [5–8], has been proved by many authors to be a powerful mathematical tool to deal various kinds of linear and nonlinear problems [9–12]. In recent years, it was also used to deal with inverse problems. In [13], using VIM, the authors discussed a one-dimensional parabolic equation with an unknown parameter $p(t)$ and a constant coefficient q of the first order term. In [14,15], the authors solved high dimensional semi-linear inverse parabolic equation on an unbounded domain using VIM. In [16], the authors identified an unknown coefficient of the first order term of the reaction-diffusion equation using He's VIM. In this paper, we will use He's VIM to solve the inverse problem (1.1a)-(1.2) with variable coefficients $q_j(t)$ $(j=1,\cdots,n)$ and unknown coefficient $p(t)$ in a high dimensional bounded domain. In the end, we test the effect of VIM when the thermal energy measurement $a(t)$ has noise $\epsilon(t)$. We show the stability of this method.

The convergence of VIM was discussed in [17].

2 The variational iteration method

We simplify (1.1a) from the following transformation

$$v(x,t)=u(x,t)r(t), \quad r(t)=\exp\Big(-\int_0^t p(s)ds\Big). \tag{2.1}$$

Then

$$u(x,t)=\frac{v(x,t)}{r(t)}, \quad p(t)=-\frac{r'(t)}{r(t)}. \tag{2.2}$$