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A Time Second-Order Mass-Conserved Implicit-Explicit Domain Decomposition Scheme for Solving the Diffusion Equations

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Abstract. In the paper, a new time second-order mass-conserved implicit/explicit domain decomposition method (DDM) for the diffusion equations is proposed. In the scheme, firstly, we calculate the interface fluxes of sub-domains from the obtained solutions and fluxes at the previous time level, for which we apply high-order Taylor's expansion and transfer the time derivatives to spatial derivatives to improve the accuracy. Secondly, the interior solutions and fluxes in sub-domains are computed by the implicit scheme and using the relations between solutions and fluxes, without any correction step. The mass conservation is proved and the convergence order of the numerical solutions is proved to be second-order in both time and space steps. The super-convergence of numerical fluxes is also proved to be second-order in both time and space steps. The scheme is stable under the stable condition $r \leq 3/5$. The important feature is that the proposed domain decomposition scheme is mass-conserved and is of second order convergence in time. Numerical experiments confirm the theoretical results.

AMS subject classifications: 65M06, 65M12, 65M55, 76S05

Key words: Diffusion equations, time second-order, mass-conserved, implicit-explicit domain decomposition, interface fluxes.

1 Introduction

Time dependent diffusion equations are widely used in science and engineering (see, for example, [1, 2], etc.). Due to the computational complexities and huge computational costs in applications, parallel computing based on domain decomposition methods

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is playing an important role in scientific computation. The non-overlapping implicit-explicit domain decomposition methods, explicitly calculating the interface values of sub-domains and implicitly solving interior values, were developed (see [3–8, 10–12]). However, these domain decomposition methods did not satisfy mass conservation. Developing mass-conserved domain decomposition schemes is being greatly required in applications, especially in the engineering. [13] gave a scheme of the block-centered finite difference domain decomposition procedure and proved the unconditional stability, mass conservation and second-order accuracy in the space step. But the convergence order of the method is only first order in time step.

In the paper, for improving the accuracy of the mass-conserved schemes, we propose a new time second-order mass-conserved implicit-explicit domain decomposition scheme for the diffusion equations with Neumann boundary conditions, based on the block-centered finite differences. In the scheme, we take the two-step technique to solve the solutions without any correction step. Firstly, the interface fluxes of sub-domains are explicitly calculated from the obtained solutions and fluxes at the previous time level, where we approximate the interface fluxes by applying high-order Taylor's expansion and transferring the time derivatives to spatial derivatives through the original equations to improve its accuracy and its stability condition. Secondly, the interior solutions and fluxes are computed by the implicit scheme in sub-domains using the relation between solutions and fluxes. The new and important feature is that the proposed domain decomposition scheme is mass-conserved and is of second order accuracy in time step. We prove the proposed scheme satisfies mass conservation. We analyze theoretically its stability and convergence and obtain that the scheme is stable under the condition

$$r=a\frac{\tau}{h^2}\leq\frac{3}{5}$$

and the error estimate of the numerical solutions is second order in both time and space steps. We also prove the super-convergence of numerical fluxes to have second order convergence in both time and space steps. Numerical experiments are given to confirm our theoretical results.

The paper is divided into five sections. We give numerical scheme and the algorithm of the time second-order mass-conserved implicit-explicit domain decomposition scheme in Section 2. We analyze the mass conservation, stability and convergence in Section 3 and Section 4. The numerical experiments are given in Section 5. Finally, conclusions are addressed in Section 6.

2 Diffusion equations and the time second-order mass-conserved domain decomposition scheme

We consider the one dimensional diffusion equations,