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A Mortar Method Using Nonconforming and Mixed Finite Elements for the Coupled Stokes-Darcy Model

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Abstract. In this work, we study numerical methods for a coupled fluid-porous media flow model. The model consists of Stokes equations and Darcy's equations in two neighboring subdomains, coupling together through certain interface conditions. The weak form for the coupled model is of saddle point type. A mortar finite element method is proposed to approximate the weak form of the coupled problem. In our method, nonconforming Crouzeix-Raviart elements are applied in the fluid subdomain and the lowest order Raviart-Thomas elements are applied in the porous media subdomain; Meshes in different subdomains are allowed to be nonmatching on the common interface; Interface conditions are weakly imposed via adding constraint in the definition of the finite element space. The well-posedness of the discrete problem and the optimal error estimate for the proposed method are established. Numerical experiments are also given to confirm the theoretical results.

AMS subject classifications: 65N12, 65N30, 65N55, 76D07, 76S05

Key words: Mortar method, nonconforming element, mixed element, inf-sup condition, Stokes equations, Darcy's equations, error estimate.

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1 Introduction

Nowadays, there are increasing works on coupled fluid-porous media flow models because of their wide applications in various engineering problems involving the interactions between surface flow and subsurface flow [1, 2, 6, 10–12, 14, 15, 17–23, 25–29, 31, 33,

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35,36]. These models are usually composed by Navier-Stokes or Stokes equations for the fluid flow motion and Darcy or Brinkman equation [37,38] for the porous media flow motion, plus interaction laws on the interface between the fluid subdomain and the porous media subdomain. The mathematical models are typical multi-domain models with multiphysics. Numerical methods for these problems are not easy because the heterogeneous differential equations are coupled together and the variables across the interface may be discontinuous.

We consider a particular fluid-porous media flow model, in which, the governing equation for the fluid flow is Stokes equations; the equation for the porous media flow is Darcy's law; the interface conditions consist of the continuity of normal velocities and normal stresses, and the so called Beavers-Joseph-Saffman condition [3,34]. Let us firstly review some related papers which motivate our work. We note that most recent studies are originated from two independent papers [17,29]. In [17], Discacciati et al. proposed an iterative subdomain method based on conforming quadratic element discretization for a second order elliptic problem in the porous domain and conforming Taylor-Hood element discretization in the fluid domain. In [29], Layton et al. treated one interface variable as a Lagrange multiplier and proved the existence and uniqueness of the weak solution. Then they proposed a finite element method for approximating the coupled system. Subsequent works are explored by several groups. Galvis and Sarkis proposed a mortar element method [20] based on Layton's formulation. In [10] and [33], the authors discussed the DG discretizations with non-matching meshes. Bernardi et al. developed a mortar finite element method in [6]. In their method, the finite elements in both subdomains are conforming and the jumps of the functions across subdomain interface are constrained to be orthogonal to a discrete test space on the interface.

In this work, our aim is developing a mortar method using nonconforming and mixed elements to discretize the coupled Stokes-Darcy problem. Specifically, nonconforming Crouzeix-Raviart elements and Raviart-Thomas elements are applied for the Stokes equations and the Darcy's equations respectively, constraints for interface conditions are imposed in the definition of the finite element spaces. The finite elements used in this paper are different from those used in [6,18], where both of them considered conforming finite elements for the subdomain problems. In [18], the Stokes and Darcy problems are independently solvable for a given interfacial pressure, and a coupled system on the interface was derived from the continuity of the normal velocity; the authors also used Lagrange multiplier method to treat the defective boundary condtions and analysed the combined inf-sup condition by using the nested saddle point theory [9]. Compared with the typical approaches for the coupled multi-domain model or the mortar finite element methods for the single PDE models, the advantages of our mortar method exist in several aspects. Firstly, the proposed mortar finite element method, different from the approaches in [20,27], can avoid using the interface Lagrange multiplier [29] and allows the meshes in different subdomains are nonmatching. Secondly, as we use nonconforming Crouzeix-Raviart elements together with the mixed finite elements, the finite element pairs can relax the high-order continuity conditions which are required in the construction of the