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Nonlinear Axisymmetric Deformation Model for Structures of Revolution

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Abstract. An axisymmetric formulation for modeling three-dimensional deformation of structures of revolution is presented. The axisymmetric deformation model is described using the cylindrical coordinate system. Large displacement effects and material nonlinearities and anisotropy are accommodated by the formulation. Mathematical derivation of the formulation is given, and an example is presented to demonstrate the capabilities and efficiency of the technique compared to the full three-dimensional model.

AMS subject classifications: 74B20, 74G15

Key words: Modeling, axisymmetry, nonlinear elasiticity, large deformations, numerical simulations.

1 Introduction

A mathematical model is a way of describing a realistic phenomenon by mathematical tools. Many of these phenomena originate from physical problems. Most physical problems are naturally formulated as initial boundary value problems in three-dimensional domains (3D). With today's techniques, 3D computations are still very expensive due to the number of unknowns and the complexity of the governing partial differential equations. The geometry of the domain is also an important source of algorithmic complexity, due to the representation of the surface and the mesh design inside the domain. Reducing the analysis of a 3D structure to two-dimensional domains (2D) provides a great convenience and efficiency compared to the full 3D analysis. In some cases, a simplified 2D model can be obtained from the 3D model by, for instance, neglecting or integrating with respect to one of the domain dimensions. This leads to only a 2D approximation model for the full 3D model. On the other hand, an intermediate situation between the full 3D geometry and the plane 2D reduction

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can be obtained. In order to perform such a reduction, however, the structure should have an axisymmetric shape. Moreover, additional assumptions of axisymmetry on the material parameters, the data and the solution are required (this is often the case for physical or mechanical systems of equations). If such assumptions of axisymmetry hold, then the resulting 2D model is called an axisymmetric model. Besides its instructional value, the treatment of axisymmetric structures has considerable practical interest in aerospace, civil, mechanical and nuclear engineering.

In the literature, a large number of axisymmetric models have been proposed. Among many other studies, for instance, Jeon et al. [12] proposed an axisymmetric model of the dome tendons in nuclear containment building. An axisymmetric finite element model has been developed by Suzuki and Maruyama [19] to evaluate earthquake responses of seismically isolated tunnels. Gallouet and Herbin [7] developed an axisymmetric cell centered finite volume scheme in order to numerically simulate the diffusion and assimilation by photosynthesis of CO_2 within a leaf. Deparis [3] studied numerical methods for an axisymmetric problem of fluid-structure interface with application to blood flow. In the framework of continuum mechanics, Wang Min and Tian You [20] developed an elastic axisymmetric model for quasi-crystal cubes. Eftaxiopoulos and Atkinson [4] treated an axisymmetric anisotropic elastic model for the angioplastic balloon. Bernardi et al. [2] developed the axisymmetric deformation model in the case of linear elasticity and small displacements.

In the present paper, we study the deformation of a structure of revolution in the framework of nonlinear elasticity and large deformations. The considered 3D computation domain is supposed invariant by rotation around an axis. Such a domain is generated by rotating a 2D set, the meridian domain, around the axis. Under some assumptions of axisymmetry on the material constitutive law and the loading, we derive the axisymmetric model as both a nonlinear system of partial differential equations and a variational problem written on the 2D meridian domain. The constitutive law can be nonlinear and anisotropic involving variable fiber direction through the material. Moreover, the main advantage of the proposed model is the fact that the deformation of the 2D meridian domain is done in the 3D space, hence allowing twisting during deformation. Therefore, the axisymmetric model considered in this paper provides a bridge to the treatment of three-dimensional nonlinear anisotropic elasticity. It is worth mentioning that in the linear framework, a structure of revolution under non-axisymmetric loading can be treated by a Fourier decomposition method. This involves decomposing the load into a Fourier series in the circumferential direction, calculating the response of the structure to each harmonic term retained in the series, and superposing the results, see Bernardi et al. [2]. The axisymmetric problem considered in this paper may be viewed as computing the response to the zero-th harmonic. This superposition technique, however, is limited to linear problems.

The main goal of the present paper is to derive and numerically validate the nonlinear axisymmetric deformation model. However, the problem of existence and uniqueness of solutions of the proposed axisymmetric model is very hard and is not addressed in the present paper. In some particular simplified similar problems, closed