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Investigation of Turbulent Transition in Plane Couette Flows Using Energy Gradient Method

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Abstract. The energy gradient method has been proposed with the aim of better understanding the mechanism of flow transition from laminar flow to turbulent flow. In this method, it is demonstrated that the transition to turbulence depends on the relative magnitudes of the transverse gradient of the total mechanical energy which amplifies the disturbance and the energy loss from viscous friction which damps the disturbance, for given imposed disturbance. For a given flow geometry and fluid properties, when the maximum of the function K (a function standing for the ratio of the gradient of total mechanical energy in the transverse direction to the rate of energy loss due to viscous friction in the streamwise direction) in the flow field is larger than a certain critical value, it is expected that instability would occur for some initial disturbances. In this paper, using the energy gradient analysis, the equation for calculating the energy gradient function K for plane Couette flow is derived. The result indicates that K reaches the maximum at the moving walls. Thus, the fluid layer near the moving wall is the most dangerous position to generate initial oscillation at sufficient high Re for given same level of normalized perturbation in the domain. The critical value of K at turbulent transition, which is observed from experiments, is about 370 for plane Couette flow when two walls move in opposite directions (anti-symmetry). This value is about the same as that for plane Poiseuille flow and pipe Poiseuille flow (385-389). Therefore, it is concluded that the critical value of K at turbulent transition is about 370-389 for wall-bounded parallel shear flows which include both pressure (symmetrical case) and shear driven flows (anti-symmetrical case).

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1 Introduction

Although more than a century has passed since the pioneering work of Reynolds (1883) was done, flow transition from laminar flow to turbulence is still not completely understood [1–5]. In practice, the understanding of turbulence transition and generation has great significance for basic sciences and many engineering fields. This issue is intricately related to the instability problem of the base flow subjected to some imposed disturbances [1,2].

In the past, several stability theories have been developed to describe the mechanism of flow instability. These are: the linear stability theory, which goes back to Rayleigh (1880) is a widely used method and has been applied to several problems [6]. For Taylor-Couette flow and Rayleigh-Bernard convective problem, it agrees well with experimental data. However, this theory fails when used for wall-bounded parallel flows such as plane Couette flow, plane Poiseuille flow and pipe Poiseuille flow; the energy method (Orr, 1907), which is based on the Reynolds-Orr equation is another mature method for estimating flow instability [7]. However, agreement could not be obtained between the theoretical predictions and the experiment data; the weakly nonlinear stability theory (Stuart, 1971) emerged in 1960's and the application is very limited (see [8]); the secondary instability theory (Herbert et al. 1988), which was developed most recently, explains some of flow transition phenomena (mainly for the boundary layer flow) better than the other earlier theories (see [9]). However, there are still significant discrepancies between the predictions obtained using this method and experimental data; particularly at transition.

Studies for parallel flows have attracted many scientists with great concern. For these parallel flows, it is observed from experiments that there is a critical Reynolds number Re_c below which no turbulence can be sustained regardless of the level of imposed disturbance. For the pipe Poiseuille flow, this critical value of Reynolds number is about 2000 from experiments [10,11]. Above this Re_c, the transition to turbulence depends to a large extent on the initial disturbance to the flow. For example, experiments showed that if the disturbances in a laminar flow can be carefully avoided or considerably reduced, the onset of turbulence can be delayed to Reynolds number up to $\mathrm{Re}=\mathcal{O}(10^5)$ [12]. Experiments also showed that for $\mathrm{Re}>\mathrm{Re}_c$, only when a threshold of disturbance amplitude is reached, can the flow transition to turbulence occur [13]. Trefethen et al. suggested that the critical amplitude of the disturbance leading to transition varies broadly with the Reynolds number and is associated with an exponent rule of the form, $A\propto\mathrm{Re}^\gamma$ [12]. The magnitude of this exponent has significant implication for turbulence research [12]. Chapman, through a formal asymptotic analysis of the Navier-Stokes equations (for $\mathrm{Re}\to\infty$), found $\gamma=-3/2$ and $-5/4$ for plane Poiseuille flow with streamwise mode and oblique mode, respectively, with generating a secondary instability, and $\gamma=-1$ for plane Couette flow with above both modes. He also examined the boot-strapping route to transition without needing to generate a secondary instability, and found $\gamma=-1$ for both plane Poiseuille flow and plane Couette flow [14]. Recently, Hof et al. [15] used pulsed injection disturbances in