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On Higher Order Pyramidal Finite Elements

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Abstract. In this paper we first prove a theorem on the nonexistence of pyramidal polynomial basis functions. Then we present a new symmetric composite pyramidal finite element which yields a better convergence than the nonsymmetric one. It has fourteen degrees of freedom and its basis functions are incomplete piecewise triquadratic polynomials. The space of ansatz functions contains all quadratic functions on each of four subtetrahedra that form a given pyramidal element.

AMS subject classifications: 65N30

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1 Introduction

Pyramidal mortar elements are very useful tools for making face-to-face connections between tetrahedral and block elements in the finite element method (see Fig. 1). This often occurs in joining tetrahedral meshes with hexahedral ones, which is common in many practical applications, where only part of the domain can be decomposed into block elements and the remainder of the domain, often near the boundary, is decomposed into tetrahedral elements. The first mortar elements were proposed by Zlámal. In [7], Zlámal introduced two kinds of triangular elements that enable us to connect the standard linear elements with the Hermite cubic elements.

Two types of composite incomplete trilinear and triquadratic pyramidal mortar elements with five (cf. Fig. 1) and thirteen degrees of freedom are presented in [6].

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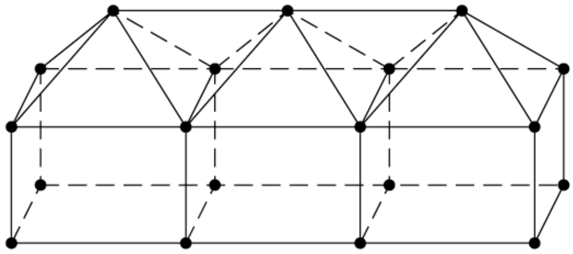

Figure 1: Mortar elements with 5 degree of freedoms.

Their basis functions are defined on the pyramidal element which is composed of two tetrahedra. This causes an artificial anisotropy in solving isotropic problems (compare with [4]). In [5] pyramidal elements composed of four tetrahedra are introduced which eliminate this artificial anisotropy. They also have five and thirteen degrees of freedom. Moreover, numerical results given in [5] indicate that an improvement of the convergence is obtained for both types of symmetric elements over the non-symmetric ones.

Wieners in [6] points out that it can be proven that it is not possible to define polynomial basis functions that are linear (or quadratic) on all the triangular faces and bilinear (or biquadratic) on the base of the pyramidal element. We will improve Wieners' result. We also develop a new composite piecewise triquadratic pyramidal element. To accomplish this we add a node at the center of the base of the element, thereby creating an element with fourteen degrees of freedom. This new pyramidal element, having nine nodes on the base, allows for a better connection to the common twenty-seven node triquadratic block element which has nine nodes on each face. Its space of ansatz functions contains all quadratic polynomials on each tetrahedron (from Figs. 2, 3 and 4).

This paper is organized as follows: in Section 2, we discuss the nonexistence of polynomial basis functions on a pyramidal element. We prove that there exists no continuously differentiable function on the pyramid (see Fig. 2), which would be linear on its four triangular faces and bilinear, but not linear, on its rectangular base. In Section 3.1 we present the basis functions for the two-tetrahedral composition of the pyramidal element (see Fig. 3) and prove that these functions meet the required criteria for basis functions. In Section 3.2 we take the average of these basis functions with their mirror images in order to derive a set of basis functions well defined on the four-tetrahedral composition of the pyramid (see Fig. 4). Finally, in Section 4 we present numerical results for both the two- and four-tetrahedral compositions.

2 Nonexistence of pyramidal polynomial basis functions

Let Co stand for the convex hull and let

$$\widehat{K} = \mathrm{Co}\{\widehat{A}_0, \widehat{A}_1, \widehat{A}_2, \widehat{A}_3, \widehat{A}_4\} = \mathrm{Co}\{(0,0,0), (1,0,0), (1,1,0), (0,1,0), (0,0,1)\},$$