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REVIEW ARTICLE

A Brief Review of Elasticity and Viscoelasticity for Solids

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Abstract. There are a number of interesting applications where modeling elastic and/or viscoelastic materials is fundamental, including uses in civil engineering, the food industry, land mine detection and ultrasonic imaging. Here we provide an overview of the subject for both elastic and viscoelastic materials in order to understand the behavior of these materials. We begin with a brief introduction of some basic terminology and relationships in continuum mechanics, and a review of equations of motion in a continuum in both Lagrangian and Eulerian forms. To complete the set of equations, we then proceed to present and discuss a number of specific forms for the constitutive relationships between stress and strain proposed in the literature for both elastic and viscoelastic materials. In addition, we discuss some applications for these constitutive equations. Finally, we give a computational example describing the motion of soil experiencing dynamic loading by incorporating a specific form of constitutive equation into the equation of motion.

AMS subject classifications: 93A30, 74B05, 74B20, 74D05, 74D10

Key words: Mathematical modeling, Eulerian and Lagrangian formulations in continuum mechanics, elasticity, viscoelasticity, computational simulations in soil, constitutive relationships.

1 Introduction

Knowledge of the field of continuum mechanics is crucial when attempting to understand and describe the behavior of materials that completely fill the occupied space and thus act like a continuous medium. There are a number of interesting applications where modeling of elastic and viscoelastic materials is fundamental. One interest in particular is in describing the response of soil which experiences some sort of impact.

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This may result from buildings falling or being imploded, or even an intentionally introduced impact as part of land mine detection efforts (see [54, 56]). The chief interest is in determining what would happen to buried objects given a particular surface impact. In the case of a building implosion, there are concerns for buried infrastructure such as tunnels, pipes, or nearby building infrastructure. Investigations can be carried out to determine the likely forces on these buried objects to ensure that the force delivered into the soil will not damage other infrastructure. When detecting land mines, the methodology developed in the papers cited uses an impact on the ground to create Rayleigh surface waves that are subsequently changed upon interacting with a buried mine; this change in wave form might be detected through electromagnetic or acoustic means. Creating a model that accurately describes these Rayleigh waves is key to modeling and understanding the buried land mine situation. In both of these examples, one must study the soil properties, determine a valid constitutive relationship for the soil, and verify the accuracy of the model. One can then use the models to predict the results from different forces, soil properties, etc. Another application is the non-invasive detection of arterial stenosis (e.g., see [1, 3, 11, 38, 50]). In this study, blockages in the artery create turbulence in the blood flow, which then generates an acoustic wave with a normal and shear component. The acoustic wave propagates through the chest cavity until it reaches the chest wall, where a series of sensors detect the acceleration of the components of the wave. The data from the sensors can then be used to quickly determine the existence and perhaps the location of the blockages in the artery. This technique is inexpensive and non-invasive. For such a technology to be feasible, a mathematical model that describes the propagation of the acoustic wave from the stenosis to the chest wall will be necessary to correctly detect the location of a blockage.

The goal of this paper is to provide a brief introduction of both elastic and viscoelastic materials for those researchers with little or no previous knowledge on continuum mechanics but who are interested in studying the mechanics of materials. The materials that we are considering are simple (for example the stress at a given material point depends only on the history of the first order spatial gradient of the deformation in a small neighborhood of the material point and not on higher order spatial gradients) and non-aging (the microscopic changes during an experiment can be neglected in the basic model). Our presentation is part tutorial, part review but not a comprehensive survey of a truly enormous research literature. We rely on parts of the standard literature and discuss our view of generally accepted concepts. We present a discussion of topics we have found useful over the past several decades; hence approximately 20% of references are work from our group. We have not meant to ignore major applications in the many fine contributions of others; rather our presentation reflects a certain level of comfort in writing about efforts on which we have detailed knowledge and experience.

The introductory review is outlined as follows: in Section 2 some basic terminology (such as stress and strain) and relationships (e.g., the relationship between strain and displacement) of continuum mechanics are briefly described. In addition, we give