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Hill-Climbing Algorithm with a Stick for Unconstrained Optimization Problems

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Abstract. Inspired by the behavior of the blind for hill-climbing using a stick to detect a higher place by drawing a circle, we propose a heuristic direct search method to solve the unconstrained optimization problems. Instead of searching a neighbourhood of the current point as done in the traditional hill-climbing, or along specified search directions in standard direct search methods, the new algorithm searches on a surface with radius determined by the motion of the stick. The significant feature of the proposed algorithm is that it only has one parameter, the search radius, which makes the algorithm convenient in practical implementation. The developed method can shrink the search space to a closed ball, or seek for the final optimal point by adjusting search radius. Furthermore our algorithm possesses multi-resolution feature to distinguish the local and global optimum points with different search radii. Therefore, it can be used by itself or integrated with other optimization methods flexibly as a mathematical optimization technique. A series of numerical tests, including high-dimensional problems, have been well designed to demonstrate its performance.

AMS subject classifications: 90C56, 90C30, 56K05

Key words: Direct search algorithm, stick hill-climbing algorithm, search radius.

1 Introduction

For simplicity, we limit our discussion to the following unconstrained optimization problem

$$\max_{x\in\Omega} f(x), \quad \Omega\subset\mathbb{R}^n,$$

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where $f:\mathbb{R}^n \to \mathbb{R}$. The following statements are also suitable to find the minima of objective function with constraint conditions. There have been innumerable approaches developed to solve these optimization problems. These methods usually can be divided into two types, derivative-based methods and derivative-free methods, depending on whether they use derivative information or not. The derivative-based approaches appeal to the Taylor's series expansion of the objective function [1–3]. For example, the steepest descent method assumes the availability of first derivatives and uses the first-order Taylor polynomial to construct local linear approximations of f. Newton's method assumes the availability of first and second derivatives and uses the second-order Taylor polynomial to construct local quadratic approximations of f. However, for a variety of reasons there have always been many instances where (at least some) derivatives are unavailable or unreliable, or finite-difference derivative approximation is unavailable or available at a prohibitive cost. Some of the reasons contain increasing complexity in mathematical modeling, higher sophistication of scientific computing, an abundance of legacy codes, and data science [4]. Nevertheless, under such circumstances it may still be desirable to carry out optimization. It follows that the derivative-free optimization technique is required.

The derivative-free optimization is an area of long history and current rapid growth in the scientific and engineering communities. The derivative-free algorithms can mainly be classified as direct and model-based [5]. Direct algorithms usually determine search directions by evaluating the function f directly, whereas model-based algorithms construct and utilize a surrogate model of f to guide the search process. Recently developed methods based trust-region using interpolation model are in this category [6–9]. In practical implementation, heuristic algorithms, such as simulated annealing, genetic algorithm, neural networks, and deep learning [10, 11], have been also developed to solve derivative-free optimization. Here we focus our attention on the direct search algorithms.

The term "direct search", firstly coined by Hooke and Jeeves in 1961 year [12], is used to describe a sequential examination of trial generated by a certain strategy. From a modern viewpoint, the direct search methods neither compute nor approximate derivatives [13]. A popular direct search method is the Nelder-Mead simplex algorithm [14]. The algorithm starts with a set of points that form a simplex. In each iteration, the objective function values at the corner points of the simplex determine the worst corner point. The algorithm attempts to replace the worst point by introducing a new vertex by a reflection, an expansion, or a contraction operator that results in a new simplex. Another class of direct search methods are the directional direct-search methods, including the original Hooke-Jeeves algorithm [12], the coordinate-, or compass-search methods, pattern-search methods [4], generalized pattern search method [15], and generating set search method [16]. A multidirectional search algorithm, regarded as both a directional and a simplicial direct search method, has been also proposed by Dennis and Torczon [17]. The common ground of these directional direct-search methods is that they incorporate some mechanism to choose ascend directions and search along these directions with an appropriate step length from the current iterate for a new iterate with a higher function value.