Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **9**, No. 2, pp. 250-271

DOI: 10.4208/aamm.2014.m888
April 2017

New Conservative Finite Volume Element Schemes for the Modified Regularized Long Wave Equation

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Received 17 December 2014; Accepted (in revised version) 19 April 2016

Abstract. In this paper, we propose a new energy-preserving scheme and a new momentum-preserving scheme for the modified regularized long wave equation. The proposed schemes are designed by using the discrete variational derivative method and the finite volume element method. For comparison, we also propose a finite volume element scheme. The conservation properties of the proposed schemes are analyzed and we find that the energy-preserving scheme can precisely conserve the discrete total mass and total energy, the momentum-preserving scheme can precisely conserve the discrete total mass and total momentum, while the finite volume element scheme merely conserve the discrete total mass. We also analyze their linear stability property using the Von Neumann theory and find that the proposed schemes are unconditionally linear stable. Finally, we present some numerical examples to illustrate the effectiveness of the proposed schemes.

AMS subject classifications: 65M99, 70H06, 74J35, 74S99
Key words: Energy, momentum, mass, finite volume element method, MRLW equation.

1 Introduction

In this paper, we consider the following initial boundary value problem

$$\begin{cases} u_t+u_x+\kappa u^2u_x-\mu u_{xxt}=0, \quad a\le x\le b, \quad t>0,\\ u(x,0)=u_0(x),\\ \dfrac{\partial^j u}{\partial x^j}\Big|_{x=a}=\dfrac{\partial^j u}{\partial x^j}\Big|_{x=b}, \quad j=0,1,2, \end{cases} \tag{1.1}$$

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where κ and μ are positive constants. The equation is also known as modified regularized long wave (MRLW) equation. In fact, MRLW equation is a special case of generalized regularized long wave (GRLW) equation which has the form

$$u_t+u_x+\kappa u^p u_x-\mu u_{xxt}=0, \tag{1.2}$$

where p is a positive integer. MRLW equation is an important equation which has many applications in sciences, for example, drift waves in plasma, the Rossby waves in rotating fluids, the transverse waves in shallow water, pressure waves in liquid-gas bubble mixtures, and so on [1,2]. Their solutions are a kind of solitary waves named solitons whose amplitudes, shapes and velocities are not affected by collision.

In this paper, the discrete variational derivative method (DVDM) [3] and the finite volume element method (FVEM) are applied to study the conservative schemes of MRLW equation. The DVDM is a method of designing special numerical schemes that retain the conservation/dissipation properties of the original partial differential equations (PDEs). In this aspect, various numerical schemes were proposed and studied in the literature. For example, Furihata and Mori [4] has proposed a stable finite difference scheme for the Cahn-Hilliard equation. Koide and Furihata [6] has used the DVDM to design four conservative schemes for the regularized long wave equation. Further, Matsuo and Furihata [7] has extended the general studies to complex-valued PDEs such as the nonlinear Schrödinger equation. Recently, the method has been extended in various ways, such as Yaguchi, Matsuo and Sugihara [8] has extended the method to nonuniform grids. Matsuo and Kuramae [9,10] have further developed an alternating DVDM, and so on.

The FVEM, as a type of important numerical tool for solving differential equations, has a long history. This method is also known as a box method in some early references [11,12], or known as a generalized difference method [13,14] in China. This method has been widely used in several engineering fields, such as fluid mechanics, heat and mass transfer and petroleum engineering. Perhaps the most important property of FVEM is that it can preserve the conservation laws (mass, momentum and heat flux) on each computational element. This important property, in combination with adequate accuracy and ease of implementation, has attracted many researchers to do research in this field [15–20].

Li and Vu-Quoc [21] once said that "in some areas, the ability to preserve some invariant properties of the original differential equation is a criterion to judge the success of a numerical simulation". Thus, the main purpose of this paper is to study the conservative schemes of the MRLW equation. For MRLW equation, researchers have done a lot of work, for instance, Gardner et al. [22] has used the cubic B-spline finite element method to solve the MRLW equation. Durán and López-Marcos [5] has investigated some advantages of conservative numerical methods. Khalifa et al. [23,24] have used the finite difference method and collocation method with cubic B-splines to solve the MRLW equation. Raslan [25] has proposed a new algorithm based on the collocation method to solve the MRLW equation. Cai [26] has proposed a multi-symplectic explicit scheme for MRLW