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Hedging Game Contingent Claims with Constrained Portfolios

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Abstract. Game option is an American-type option with added feature that the writer can exercise the option at any time before maturity. In this paper, we consider the problem of hedging Game Contingent Claims (GCC) in two cases. For the case that portfolio is unconstrained, we provide a single arbitrage-free price P_0. Whereas for the constrained case, the price is replaced by an interval $[h_{low}, h_{up}]$ of arbitrage-free prices. And for the portfolio with some closed constraints, we give the expressions of the upper-hedging price and lower-hedging price. Finally, for a special type of game option, we provide explicit expressions of the price and optimal portfolio for the writer and holder.

AMS subject classifications: 60G40, 91A60

Key words: Game option, contingent claims, hedging, optimal stopping, free boundary.

1 Introduction

In [2], Kifer introduced Game option, and he also gave the expressions of the value process and optimal stopping times for the holder and writer. Then many authors continue the research, in this direction, see, e.g., [4–6, 8–11] and so on. However, most of the research focuses on the deduction of the expressions and properties of the price. In this paper, we will mainly consider the problem of hedging Game Contingent Claims (GCC).

In section 2 we briefly give some background information, including market model and some definitions. In section 3 we discuss the portfolio without constrains and point out that the upper– and lower–hedging prices both equal to a given arbitrage-free price P_0. Section 4 we begin to study portfolio constraints, where we mainly investigate the hedging problem for the GCC under general portfolio constraints, and

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give an arbitrage-free interval $[h_{low}, h_{up}]$. Section 5 is the main part of this paper. In this section we consider the constraints on portfolio and obtain the expressions of upper- and lower-hedging prices based on the introduction of an auxiliary family $\{\mathcal{M}_\nu\}_{\nu\in\mathcal{D}}$. In addition, we also point out that there exists a hedging portfolio. In section 6, we give some examples.

2 Market model

Consider the Black-Scholes market $\mathcal{M}$. That is, there is only one risky asset S and a riskless bond B. They satisfy

$$dB_t = B_t r_t dt, \qquad B_0 = 1, \tag{2.1}$$
$$dS_t = S_t[r_t dt + \sigma_t dW_t], \quad S_0 = x \in (0,\infty), \tag{2.2}$$

respectively, with W a standard Brownian motion on the complete probability space $(\Omega, \mathcal{F}, \mathbf{P})$, and we will denote its natural filtration

$$\mathcal{F}_t^W = \sigma(W_s : 0 \le s \le t), \quad 0 \le t \le T < \infty, \quad \text{by} \quad \mathbf{F} = \{\mathcal{F}_t\}_{0\le t\le T}.$$

Here the process r_t is interest rate, σ_t is volatility. We suppose that they are all $\mathbf{F}$-progressively measurable and bounded uniformly in $[0,T]\times\Omega$. Furthermore, $\sigma_t(\omega)$ will be assumed to be invertible, with $\sigma_t^{-1}(w)$ bounded in $[0,T]\times\Omega$. Let

$$\beta_t = 1/B_t = \exp\Big\{ -\int_0^t r_s\, ds\Big\}.$$

Then

$$d\beta_t S_t = \beta_t S_t \sigma_t dW_t. \tag{2.3}$$

That is, under $\mathbf{P}$ the discounted stock price $\beta_t S_t \triangleq \widetilde{S}_t$ is a martingale, so we will call $\mathbf{P}$ an equivalent martingale measure.

Definition 2.1. (1) *An* $\mathbf{F}$*-progressively measurable process* $\pi : [0,T]\times\Omega \to \mathbf{R}$ *with*

$$\int_0^t \pi_t^2 dt < \infty \,\text{a.s.},$$

is called a portfolio process. (2) *An* $\mathbf{F}$*-adapted process* $C : [0,T]\times\Omega \to \mathbf{R}$ *with increasing, right continuous paths and*

$$C_0 = 0, \quad C_T < \infty, \quad \text{a.s.},$$

is called cumulative consumption process. We will call (π, C) *portfolio consumption process.*

Definition 2.2. *For any given portfolio consumption process pair* (π, C), $x \in \mathbf{R}$, *the solution*

$$V \equiv V^{x,\pi,C},$$