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Lattice Boltzmann Simulation of Free-Surface Temperature Dispersion in Shallow Water Flows

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Abstract. We develop a lattice Boltzmann method for modeling free-surface temperature dispersion in the shallow water flows. The governing equations are derived from the incompressible Navier-Stokes equations with assumptions of shallow water flows including bed frictions, eddy viscosity, wind shear stresses and Coriolis forces. The thermal effects are incorporated in the momentum equation by using a Boussinesq approximation. The dispersion of free-surface temperature is modelled by an advection-diffusion equation. Two distribution functions are used in the lattice Boltzmann method to recover the flow and temperature variables using the same lattice structure. Neither upwind discretization procedures nor Riemann problem solvers are needed in discretizing the shallow water equations. In addition, the source terms are straightforwardly included in the model without relying on well-balanced techniques to treat flux gradients and source terms. We validate the model for a class of problems with known analytical solutions and we also present numerical results for sea-surface temperature distribution in the Strait of Gibraltar.

AMS subject classifications: 65M10, 78A48

Key words: Shallow water flows, free-surface temperature, lattice Boltzmann method, advection-diffusion equation, strait of Gibraltar.

1 Introduction

During the last years the increase of sea-surface temperature has attracted much interest in numerical methods for the prediction of its transport and dispersion. In many situations, this sea-surface temperature has detriment impact on the ecology and environment and may cause potential risk on the human health and local economy.

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Efficient and reliable estimates of impacts on the water quality due to free-surface temperature could play essential role in establishing control strategy for environmental protection. Introduction and utilization of such measures are impossible without knowledge of various processes such as formation of water flows and dispersion of sea-surface temperature. The mathematical models and computer softwares could be very helpful to understand the dynamics of both, water flow and sea-surface temperature dispersion. In this respect mathematical modeling of water flows and the processes of transport-dispersion of sea-surface temperature could play a major role in establishing scientifically justified and practically reasonable programs for long-term measures for a rational use of water resources, reduction of thermal discharge from particular sources, estimation of the impact in the environment of possible technological improvements, development of methods and monitoring facilities, prediction and quality management of the environment, etc. The success of the computational methods in solving practical problems depends on the convenience of the models and the quality of the software used for the simulation of real processes.

Clearly, the process of free-surface temperature dispersion is determined by the characteristics of the hydraulic flow and the temperature properties of the water. Thus, dynamics of the water and dynamics of the temperature must be studied using a mathematical model made of two different but dependent model variables: (i) a hydrodynamic variable defining the dynamics of the water flow, and (ii) a thermal variable defining the transport and dispersion of the temperature. In the current work, the hydrodynamic model is based on a two-dimensional shallow water equations while, a convection-diffusion equation is used for the free-surface temperature. For environmental flows, the shallow water system is a suitable model for adequately describing significant hydraulic processes. The different characteristics of thermal problems require an appropriate model to describe their dynamics, nevertheless for a wide class of thermal predictions the standard convection-diffusion equation can be used. The interaction between the two processes gives rise to a hyperbolic system of conservation laws with source terms.

Various numerical methods developed for general systems of hyperbolic conservation laws have been applied to the shallow water equations. For instance, most shock-capturing finite volume schemes for shallow water equations are based on approximate Riemann solvers which have been originally designed for hyperbolic systems without accounting for source terms such as bed frictions, eddy viscosity, wind shear stresses and Coriolis forces. Therefore, most of these schemes suffer from numerical instability and may produce nonphysical oscillations mainly because dicretizations of the flux and source terms are not well-balanced in their reconstruction. The well-established Roe's scheme [26] has been modified by Bermúdez and Vázquez [7] to treat source terms. This method has been improved by Vázquez [38] for general one-dimensional channel flows. However, for practical applications, this method may become computationally demanding due to its treatment of the source terms. Alcrudo and Garcia-Navarro [2] have presented a Godunov-type scheme for numerical solution of shallow water equations. Alcrudo and Benkhaldoun [1] have developed exact