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Boundary Vorticity Flux and Engineering Flow Management

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Abstract. To improve the performance of complex viscous engineering flows, the focus should be on local dynamics (local processes and structures) measured by the space-time derivatives of the primary-variable fields, rather than these fields themselves. In the context of optimal flow management such as optimal configuration design and flow control, the local fluid dynamics on solid wall is of most direct relevance. For large Reynolds-number flows, we show that the on-wall local dynamics is highlighted by the balance between tangential pressure gradient and vorticity creation rate at the wall (boundary vorticity flux, BVF), namely the on-wall coupling of the compressing and shearing processes. This basic concept is demonstrated by previously unpublished and newly obtained numerical examples for external and internal flows, including the role of BVF as a faithful marker of the local appearance of boundary-layer separation and wall curvature discontinuity, and the use of BVF-based formulas to optimize the integrated performance of airfoil and compressor rotor blade.

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1 Introduction

Any practical external or internal engineering flow has a set of global performances as its design objectives, e.g., the lift and drag of a wing or the pressure ratio and efficiency of a compressor, as well as the operational stability of the flow. Ever since

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Helmholtz [3] realized for the first time the crucial importance of local vortical structures measured by the vorticity in practical flows, it has now been well recognized that all global performances of engineering flows are dominated by various local dynamic structures, such as boundary layers, free shear layers, vortices, turbulent coherent structures, shock waves and other nonlinear waves. In a multi-dimensional, viscous and compressible flow, the local structures as exemplified above come from three fundamental dynamic processes [1,5]: the (transverse) *shearing process*, the (longitudinal) *compressing process* and the *thermal process*. The first two processes are the fundamental bulk dynamic processes in fluid motion. They are measured by the vorticity field and dilatation-pressure field (or other proper scalar field) and their characteristic behaviors are governed by the Reynolds number and Mach number, respectively. These two fundamental processes are inherently coupled both in the interior of the flow field via the nonlinear terms of the governing equations and on flow boundary via the adherence condition. In addition, the thermal process is inevitably involved as long as the flow is compressible. It can be conveniently measured by the entropy gradient field which is inherently coupled with both compressing and shearing processes. This is why modern physical theories, experiments and computations on complex flows have been focusing on the nonlinear evolutions and interactions of these local structures. Naturally, a deep and quantitative physical understanding of these structures and their role in global flow performance has become the very basis of *optimal flow management* including configuration design and flow control.

As seen above, any of the local structures are measured not by primary variables themselves (e.g., velocity $\boldsymbol{u}$, pressure p and entropy s) but their spatial-temporal derivatives, which appear in the local balances of mass, momentum and energy, and thereby interact each other to produce various dynamic effects, especially the aerodynamic force. Indeed, a uniform $(\boldsymbol{u}, p)$ field produces no force at all; a wing experiences a pressure force only if *p varies* over its surface. Dynamically, the pressure field itself can only be related to the kinetic energy via the Bernoulli equation, which however exists only when the flow is circulation-preserving, and merely reflects the dynamics of longitudinal (compressing) process [11] †. Circulation is no longer preserving in generic viscous shear flows.

There have been various theories on global flow performance in terms of local structures in both external and internal aerodynamics, which are systematically presented by Wu, Ma & Zhou [11], Wu, Lu & Zhuang [10] and Yang et al. [18,19]. While all these theories may enhance our physical understanding of the flow structures in their different evolution stages and their relevance to global performances, they are not equally useful in flow management, which depends directly and critically on the *local dynamics right on the solid boundaries*. A solid wall is the ultimate root of all flow structures that in turn leave signature thereon. The management of these on-wall root and signature, therefore, is of particular importance in the improvement of global flow

†In a circulation-preserving flow, the shearing process appears in a series of vorticity conservation theorems. It is coupled with the compressing process only through the vorticity-induced velocity that contributes to the kinetic energy.