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The Fundamental Secondary Instability of the Primary Mack Mode in Hypersonic Boundary Layers on Flat Plates

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Abstract. The secondary instability was investigated in high-speed boundary layers over flat plates by numerical methodology. The numerical simulation suggests that the main streaky structures in 2D hypersonic boundary transition process are resulted from the secondary instability theory caused by the primary 2D Mack mode. The secondary instability analysis found that a new family called fundamental family of solutions was found which is the least stable secondary instability when the amplitude of the primary mode instability reaches a threshold value. It is help for us to understand the fundamental breakdown in hypersonic boundary layers.

AMS subject classifications: 76E19, 76N20

Key words: Secondary instability, fundamental mode, boundary layers, hypersonic.

1 Introduction

The laminar-turbulence transition is one of the unsolved problems in the fluid mechanics. Generally, it can be found that there are λ vortices structures in the transition process. The generation of the λ vortices structures can be explored by the secondary instability theory (SIT) which was proposed by Herbert [1] in the two dimensional (2D) incompressible flows firstly. In the incompressible flow, the least stable mode is the T-S mode which is two dimensional in the planar case. when it grows to a higher enough amplitude, it tends to the saturation. At the same time, the saturated T-S mode disturbance is added to the original basic flow which is always the Blassius solution in flat plate case. Then, they become a new basic flow which is three dimensional (3D). As a result, a new unstable mode disturbance can be found in this type basic flow by the linear theory. The new mode

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often has a larger growth rate with a finite spanwise wave number. This is the secondary instability theory. In other words, the SIT can be used to understand that where the 3D structures come from since the least stable instability mode in the incompressible flow is planar. In incompressible flows, the SIT is very useful to understand the early stage of the vortices structures in the transition process. In the supersonic case which the freestream Mach number is often less than 3, the first mode which is often oblique dominates the transition process. Compared to the classical SIT, the nonlinear interaction between the oblique modes are significant in the breakdown process. In hypersonic flows, is there any secondary instability? It is a very interesting problem in the theory.

Actually, the transition in the hypersonic flow is much more complex for the existence of the multiple instability modes and the numerous nonlinear interactions. It has been very known that the linear instability modes are often multiple in a hypersonic flow. These modes are called the first mode and the higher modes. The first mode has the less frequency and often can be seen as the extension of the T-S mode instability in a hypersonic case. On the contrary, the ones of the higher modes are higher which are inviscid instability. They are caused by the hyperbolic characteristic of the pressure disturbance equation when the disturbance is supersonic relative to the basic flow essentially. In these higher modes, the least stable higher mode is the second mode which is planar. For $Ma>4$, it is known that the second mode usually has the highest amplified rate in all instability modes. For the early stage of the transition process, the development of the instability mode is exponential on account of the lower amplitude. And then when it grows to a threshold, the nonlinear interactions occur and dominate the transition process. Compared to its incompressible/subsonic counterpart, the nonlinear breakdown mechanism in hypersonic boundary layers is much more complex to be understood. Obviously, the second modes always have the highest amplified rate in the hypersonic boundary layer and then grows to the highest amplitude. However, it is planar in 2D boundary layer. It implies that it can't cause the transition by itself and there must be an extra nonlinear mechanism enhanced the growth of the oblique modes in the transition process. As the similar paradox in the incompressible flows, the secondary instabilities may be one of the candidates. However, few work focuses on this topic. As a result, the SIT should be analyzed for understanding the generation of the 3D structures in hypersonic flows.

As introduced above, the SIT can be used to understand the 3D characteristics quantitatively in the transition process. Generally, it often has two kinds of the instability modes: the fundamental mode and the subharmonic one. The subharmonic mode often has one half frequency of the primary instability mode which causes the H-type transition in flows. On the contrary, the frequency of the fundamental one is as same as the one of the primary mode which results in the K-type transition. The length characteristic of the 3D structures predicted by the Herbert [1] were identified as the relevant mechanisms of the H-/N-type and K-type transition in the experiment. It seems that the subharmonic mode dominates the flow in a quite free-stream and the fundamental one dominates the flow when the background is noisy. However, the situation may be reversed in the hypersonic flow. In the earlier stage, Ng [2] investigated the secondary instability in hy-