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A New High-Order Conservative Patched Grid Strategy Based on Collapsed Edge

Yidao Dong[1], Xiaogang Deng[1], Yaming Chen[1,2,*], Dan Xu[3] and Guangxue Wang[4]

[1] *College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, China*
[2] *College of Science, National University of Defense Technology, Changsha 410073, China*
[3] *State Key Laboratory of Aerodynamics, China Aerodynamics Research and Development Center, Mianyang 621000, China*
[4] *School of Physics, Sun Yat-sen University, Guangzhou 510275, China*

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Abstract. A new high-order conservative patched grid strategy is developed with the concept of collapsed edge. Different from the traditional patched grid algorithm, where the patched interface is a line in two-dimensional space, the new strategy employs a transition region, where a collapsed edge is located within to form four point matched blocks. The introduction of the collapsed edge ensures that the grid generation of neighbouring blocks connected by the transition region is independent. The biggest advantage of this new strategy is that conservation is satisfied automatically since the numerical flux at the collapsed edge is zero. Furthermore, when applied in the framework of high-order finite difference methods based on the coordinate transformation, no essential modifications are needed. Some typical numerical examples are simulated with the new patched grid strategy combined with the fifth-order Weighted Compact Nonlinear Schemes (WCNS) to verify the capability of this new method.

AMS subject classifications: 65N06, 65N22

Key words: Patched grid, high-order, conservative, collapsed-edge, WCNS.

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1 Introduction

Compared with the Finite Volume Method (FVM) and the Finite Element Method (FEM) based on unstructured grid, the Finite Difference Method (FDM) combined with structured grid is more attractive in terms of the computational efficiency and the manipula-

*Corresponding author.
Email: chenym-08@163.com (Y. M. Chen)

tion complexity. Furthermore, the structured grid is suitable for capturing the characteristics of flowfields, especially in the region of boundary layer. However, the structured grid is incapable of handling complex configurations and there is a great difficulty in generating multi-block grids with matched interface. Besides, in order to capture the fine flow structures, such as the vortex dominated region, it is necessary to refine the grid, thus influencing the grid distribution in other regions. To overcome these drawbacks, the patched grid is introduced. Different from the multi-block point matched grid, the only requirement of the patched grid is that two neighbouring blocks share the same patched interface. The grids in the block could be generated independently. Obviously, the patched grid is much more flexible.

Along with the flexibility of the grid generation, corresponding numerical algorithms need to be developed for the communication in the patched interface. Generally, these algorithms should be conservative, robust and accurate. So far, the majority of the patched grid algorithm is first or second order and the robustness is excellent. Therefore, great attention is concentrated on the flux conservation at the patched interface. Rai [1] was among the first to study the patched grid algorithm for the finite difference method. Walters [2] extended the algorithm to the FVM. It should be noted that the algorithm of Rai is only accurate to first order, and the stair-like effect is unavoidable when interpolating from coarse to fine grids. To eliminate this effect, Thomas et al. [3] adopted the piecewise linear reconstruction to realize the accuracy of second order. Lerat and Wu [4] proposed an unconditionally stable algorithm for the dissipative finite difference scheme. This algorithm is equal to the area-weighted interpolation of state vectors. Zhang et al. [5] employed a conservative remapping method to improve the accuracy of data exchange on the patched grid interface and applied this method to simulate the DLR-F4 wing-body combination with a mending plate patched on the wing upper surface. Recently, Rinaldi et al. [6] developed a new patched grid algorithm for the cell-centred finite volume scheme based on the concept of supermesh. With the auxiliary supermesh, connections are built between neighbouring blocks of the patched interface. The numerical flux is calculated in the supermesh and added to the original control volume accordingly.

Apart from the conservative algorithms discussed above, some non-conservative algorithms are also studied. It is well known that the accuracy as well as the robustness would be destroyed with non-conservative algorithms, especially when the patched interface is located in the vicinity of discontinuity or the large gradient region. The CHIMERA interpolation [7], the most popular non-conservative algorithm, was applied in the overlapping grid originally. Kao and Lion [8] modified the CHIMERA method and replaced the overlapping region with the newly defined unstructured grid. Wang et al. [9] analysed the conservation error of the CHIMERA method and proved that when the discontinuity was away from the interface of the overlapping region, the error is a first order term of the mesh elements size when a second order conservative scheme was used. They also pointed out that the numerical solution would be inaccurate with non-conservative methods even for the smooth flowfields. Tang and Zhou [10] studied the accuracy and the convergence characteristics of non-conservative methods systemat-