

## The Static Solution for the Layered Piezoelectric Bounded Domain with Side Face Load by the Modified SBFEM

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**Abstract.** The static response of two-dimensional horizontal layered piezoelectric bounded domain with side face load was investigated. In this paper, the modified scaled boundary finite element method (SBFEM) is provided as an effective semi-analytical methodology. The method is used to solve the static problem for the layered piezoelectric bounded domain. The scaling line definition extends the SBFEM to be more suitable for analyzing the multilayered piezoelectric bounded domain. It avoids the limitations of original SBFEM in modeling the horizontal layered bounded domain. The modified SBFEM governing equation with piezoelectric medium is derived by introducing Duality variable in the Hamilton system. This derivation technology makes the progress be concise. The novel displacement and electric governing equations of the modified SBFEM is given together by the first time. The node forces can be expressed as power exponent function with radial coordinate by introducing the auxiliary variable and using the eigenvalue decomposition. The novel modified SBFEM solution of layered bounded domain with piezoelectric medium is solved. The new power expansion function of layered piezoelectric medium with side face load is proposed. This technology significantly extends the application range of modified SBFEM. The novel treatment of side face load for the layered piezoelectric bounded domain is proposed. Numerical studies are conducted to demonstrate the accuracy of proposed technique in handling with the static problem of layered bounded domain with piezoelectric medium for side face load. The influence of the side face load type and depth are discussed in detail.

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## 1 Introduction

The static analysis of the layered bounded domain with piezoelectric medium is an attractive problem, such as the layered piezoelectric plate analysis, sensors application, aerospace panels, structural analysis and fracture mechanics. Because of the excellent piezoelectric material properties, the piezoelectric material was used to the smart structures and small order magnitude model. The widely applications and mathematical challenges characteristics of piezoelectric material has attracted many researchers to study the static analysis of layered piezoelectric model [2, 27]. Therefore, it is worthily to pay attention to the static analysis of layered bounded domain with piezoelectric medium, especially for the horizontal layered piezoelectric model. The piezoelectric material owns excellent ability in elector-mechanical convertibility field. Thus, the piezoelectric analysis becomes a favorite topic in engineering application. However, there are very few analytical solutions and numerical solutions for the horizontal layered bounded domain with piezoelectric medium. Therefore, it has more practical significance to propose a new solution for the static responses of the horizontal layered piezoelectric medium.

The piezoelectric material has excellent material properties. Thus, the electroelastic problem for layered medium has been investigated by many researchers. The piezoelectric material has many advantages, such as the high accuracy, miniaturization and sensitive characteristics. It can be applied to many different fields, such as the electroacoustic transducers, microrobot, atomic force microscope cantilevers and structural fracture mechanics [24, 55]. The piezoelectric material also was extended to model the half space domain problem [3] and the unbounded domain problem [36]. The layered piezoelectric medium widely exists in the nature. The piezoelectric composite structures, such as the layered piezoelectric plates and beams, require the efficient and accurate electromechanical model which has electric and mechanical behaviors. The piezoelectric structures can solve by the coupled equations with electric and mechanical interlaminar continuity conditions and boundary conditions [26]. Exact solutions of the flat panels and rectangular plates with the piezoelectric medium have been obtained [44, 45, 54]. Based on the three dimensional theory of elasticity, the static and free vibration of a cross-ply laminated composite plate in piezoelectric layers has been analyzed [22]. The analytical solutions can be easily solved for the simple model shapes with specified boundary conditions. Therefore, the finite element technology becomes a top priority to analyze the general piezoelectric structures. The finite element piezoelectric model which employs the Hamilton's variational principle has been analyzed [31]. According to the coupled refined high-order global-local theory, a finite element model for the sandwich beams